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Weak decays of J/ψ: the non-leptonic case

Yu-Ming Wang[1], Hao Zou[1], Zheng-Tao Wei[2], Xue-Qian Li[2], Cai-Dian Lü[1,a]

[1] Institute of High Energy Physics, P.O. Box 918(4), Beijing 100049, P.R. China
[2] Department of Physics, Nankai University, Tianjin 300071, P.R. China

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Abstract. In our previous study, we calculated the transition form factors of $J/\psi \to D_{(s)}^{(*)}$ using the QCD sum rules. Based on the factorization approximation, the form factors obtained can be applied to evaluate the weak non-leptonic decay rates of $J/\psi \to D_{(s)}^{(*)} + M$, where M stands for a light pseudoscalar or vector meson. We predict that the branching ratio for inclusive non-leptonic two-body weak decays of J/ψ, which are realized via the spectator mechanism, can be as large as 1.3×10^{-8}; in particular, the branching ratio of $J/\psi \to D_s^{*\pm} + \rho^{\mp}$ can reach 5.3×10^{-9}. Such values will be marginally accessed by the ability of BESIII, which will begin running very soon.

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1 Introduction

The decays of J/ψ are dominated by strong and electromagnetic interactions via $c\bar{c}$ annihilating into intermediate gluons and a photon in the s-channel. By contrast, weak decays, due to the smallness of the strength of the weak interaction, are rare processes. In the spectator approximation, one of the charm quark or anti-charm quark in J/ψ decays into light quarks, and the decay rate of a charm quark (anti-quark) is proportional to $G_F^2 m_c^5$ where G_F is the Fermi coupling constant. Numerically the total branching ratio of weak decays was estimated to at the order of 10^{-8} [1]. Recently, due to remarkable improvements of experimental instruments and techniques people turn their interests onto these rare processes from both the experimental [2, 3] and theoretical [4–6] sides. The forthcoming upgraded BESIII will be able to accumulate more than 10^{10} J/ψ per year [7], which makes it possible to marginally measure such weak decays in the near future. More important, such rare processes are also particularly interesting from the viewpoint of the theory. On the one hand, it may provide a further accurate examination of the mechanism, which is responsible for the hadronic transition and fully governed by non-perturbative QCD effects. One can also expect that such decays may offer a unique opportunity to probe new physics beyond the standard model [8, 9], including the minimal supersymmetric standard model, the extra dimension model, the two-Higgs doublet, topcolor-assisted technicolor model etc., in the weak decay of vector mesons. The reason is that in such rare decays, the weak coupling is rather weak and new physics may have a chance to show up.

In a previous study, we presented a detailed analysis of the semi-leptonic decays of J/ψ [6], where the branching ratios for such channels were estimated to be at order of 10^{-10} and hence almost impossible to be observed at BESIII.

The fundamental ingredients involved in the semi-leptonic processes are the transition form factors of $J/\psi \to D_{(s)}^{(*)}$, which are evaluated in terms of the three-point QCD sum rules (QCDSR) [10–12] in that work. Obviously, even though while deriving the form factors our goal was to estimate the branching ratios of semi-leptonic decays, under the factorization approximation, they can be applied to study the non-leptonic decays. Thus, we will take the step forward to investigate the exclusive non-leptonic decays with focusing on two-body processes.

In this study, we will explore the non-leptonic decays $J/\psi \to D_{(s)}^{(*)} + M$, where the final states contain a single charmed meson and a light meson M, such as π, K, ρ, K^*, etc. These weak decays are realized via the spectator mechanism, meaning that one of the charm quark (anti-charm quark) acts as a spectator. In the standard model, at the quark level, the Feynman diagrams for charm quark decay are depicted in Fig. 1. The anti-charm quark decay can be obtained analogously by exchanging $c \leftrightarrow \bar{c}$. The effective theory for hadronic weak decays have been well formulated [13]. The most difficult work is to calculate the hadronic matrix elements that are governed by the non-perturbative QCD dynamics.

Non-relativistic QCD can simplify the picture by phenomenologically handling some non-perturbative QCD effects and has been widely applied to study some decay

[a] e-mail: lucd@ihep.ac.cn

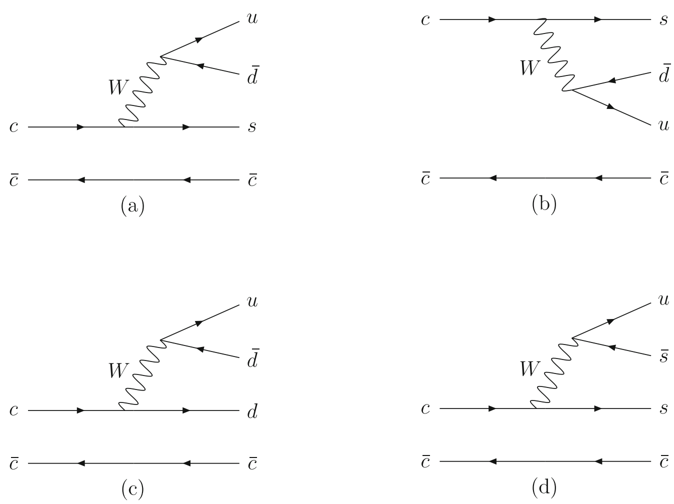

Fig. 1. Quark diagrams for non-leptonic weak decays of J/ψ. **a** represents the color allowed processes; **b** represents the color suppressed processes; **c** and **d** represent single-Cabibbo suppressed processes

modes, where heavy quarkonium is involved. However, it does not help much for the heavy-light mesons, where relativistic effects may be significant.

The first order approximation for the derivation is the factorization hypothesis, where the hadronic matrix element is factorized into a product of two matrix elements of single currents [14–19]. In this scheme, one element can be written in terms of the decay constant of the meson concerned, while the other is expressed by a few form factors according to the Lorentz structure of the current and meson (a pseudoscalar or vector, for example). The non-factorizable effects are incorporated into the effective coefficients, which are usually assumed to be universal and determined by experiment. (Only in some cases, they are perturbatively calculable. In reality, these coefficients depend on the concrete processes and differ case by case, but the variation may be not very drastic.) For the weak decays of heavy mesons, such a factorization approach is verified to work very well for the color allowed sub-processes. It is reasonable to believe that this conjecture would be valid for J/ψ, at least for the processes in which the color allowed sub-processes dominate. Thus, the study of two-body non-leptonic decays offers an ideal opportunity to test the factorization hypothesis in the heavy-quarkonium system, and this test may be more appealing than in decays of $D^{(*)}$, because J/ψ contains two heavy constituents. Moreover, it is of great importance to discriminate various theoretical tools for the evaluations of transition form factors.

The structure of this paper is as follows. In Sect. 2, the factorization approach for the non-leptonic decays is introduced and the formulas are given. In Sect. 3, after displaying the input involved in this work explicitly, the numbers of the branching fractions for various $J/\psi \to D^{(*)}_{(s)} + M$ modes are presented and comparisons of our numerical results with those estimated in other theoretical models are also investigated at length in this section. The final section is devoted to a discussion and conclusions. It is noted that since most of the form factors applied in this work were obtained in our previous work, we generally refer the readers to it for some details of the derivation and for how to achieve the numerical values.

2 Non-leptonic decays $J/\psi \to D_{(s)} + M$ in the factorization approach

For the non-leptonic weak decays of $J/\psi \to D_{(s)} + M$, the standard method is integrating out the heavy W-boson and obtaining a low energy effective Hamiltonian for c quark decay, which is given by

$$\mathcal{H}_{\text{eff}}(c \to qu\bar{q}') = \frac{G_{\text{F}}}{\sqrt{2}} V^*_{cq} V_{uq'} (C_1 Q_1 + C_2 Q_2) \,, \qquad (1)$$

where $q(q')$ represents the down type quarks s and d; $V^*_{cq}(V_{uq'})$ are CKM matrix elements; and the operators Q_1 and Q_2 are,

$$\begin{aligned} Q_1 &= \bar{q}_\alpha \gamma_\mu (1-\gamma_5) c_\alpha \bar{q}'_\beta \gamma^\mu (1-\gamma_5) u_\beta \,, \\ Q_2 &= \bar{q}_\alpha \gamma_\mu (1-\gamma_5) c_\beta \bar{q}'_\beta \gamma^\mu (1-\gamma_5) u_\alpha \,. \end{aligned} \qquad (2)$$

It should be pointed out that the penguin operators are neglected in this work due to the smallness of the Wilson coefficients for such operators, which also indicates that CP symmetry is well respected within the accepted assumption.

With the free quark decay amplitude, we can proceed to calculate the transition amplitudes for $J/\psi \to D + M$ at hadron level, which can be obtained by sandwiching the free-quark operators between the initial and final mesonic states. Consequently, the hadronic matrix elements $\langle DM|Q_i|J/\psi\rangle$, which depend on the strong interactions, need to be computed. The evaluation is indeed the main challenge in heavy flavor physics due to our poor knowledge with respect to non-perturbative QCD. Owing to the painstaking efforts in the theory, several systemic approaches for hadronic B decays have been explored based on the expansion in small parameters [20]. However, a systematic theoretical method for open-charm decays is still not available yet, due to the fact that the accessible charm quark mass is not so heavy in reality. As a first order approximation, we may apply the vacuum saturation approximation to factorize the four-quark operator matrix elements $\langle DM|Q_i|J/\psi\rangle$. The consistency of the theoretical prediction with data (which may be available in the future) will serve as an examination of such approximation in the heavy-quarkonium system as mentioned in the introduction. To be more specific, the factorization ansatz [14] states that the matrix elements can be factorized into a product of two single matrix elements of currents $\langle M|J_1|0\rangle\langle D|J_2|J/\psi\rangle$, where one is parameterized by the decay constant of the emitted light meson and the other is represented by the form factors responsible for the transition of J/ψ into the recoiled charmed meson.

The decay constants for pseudoscalar (P) and vector (V) mesons are defined as follows:

$$\langle P(q)|A_\mu|0\rangle = -\mathrm{i} f_P q_\mu\,,$$
$$\langle V(q,\epsilon)|V_\mu|0\rangle = f_V m_V \epsilon^*_\mu\,, \quad (3)$$

where the axial vector current A_μ represents $\bar{q}_1\gamma_\mu\gamma_5 q_2$ and the vector current V_μ represents $\bar{q}_1\gamma_\mu q_2$; ϵ is the polarization vector of V. The matrix elements $\langle D|\bar{q}\gamma_\mu(1-\gamma_5)c|J/\psi\rangle$ are parameterized in terms of various form factors as follows [6]:

$$\begin{aligned}
&\langle D(p_2)|\bar{q}\gamma_\mu(1-\gamma_5)c|J/\psi(\epsilon_\psi,p_1)\rangle \\
&= -\epsilon_{\mu\nu\alpha\beta}\epsilon^\nu_\psi p_1^\alpha p_2^\beta \frac{2V(q^2)}{m_\psi+m_D} \\
&+\mathrm{i}(m_\psi+m_D)\left[\epsilon_{\psi\,\mu}-\frac{\epsilon_\psi\cdot q}{q^2}q_\mu\right]A_1(q^2) \\
&+\mathrm{i}\frac{\epsilon_\psi\cdot q}{m_\psi+m_D}A_2(q^2)\left[(p_1+p_2)_\mu-\frac{m_\psi^2-m_D^2}{q^2}q_\mu\right] \\
&+2\mathrm{i}m_\psi\frac{\epsilon_\psi\cdot q}{q^2}q_\mu A_0(q^2)\,, \qquad (4)
\end{aligned}$$

$$\begin{aligned}
&\langle D^*(\epsilon_{D^*},p_2)|\bar{q}\gamma_\mu(1-\gamma_5)c|J/\psi(\epsilon_\psi,p_1)\rangle \\
&= -\mathrm{i}\epsilon_{\mu\nu\alpha\beta}\epsilon^\alpha_\psi\epsilon^{*\beta}_{D^*}\left[\left(p_1^\nu+p_2^\nu-\frac{m_\psi^2-m_{D^*}^2}{q^2}q^\nu\right)\tilde{A}_1(q^2)\right. \\
&\left.+\frac{m_\psi^2-m_{D^*}^2}{q^2}q^\nu\tilde{A}_2(q^2)\right]+\frac{\mathrm{i}}{m_\psi^2-m_{D^*}^2}\epsilon_{\mu\nu\alpha\beta}p_1^\alpha p_2^\beta \\
&\times\left[\tilde{A}_3(q^2)\epsilon^\nu_\psi\epsilon^*_{D^*}\cdot q-\tilde{A}_4(q^2)\epsilon^{*\nu}_{D^*}\epsilon_\psi\cdot q\right] \\
&+(\epsilon_\psi\cdot\epsilon^*_{D^*})\left[-(p_{1\mu}+p_{2\mu})\tilde{V}_1(q^2)+q_\mu\tilde{V}_2(q^2)\right] \\
&+\frac{(\epsilon_\psi\cdot q)(\epsilon^*_{D^*}\cdot q)}{m_\psi^2-m_{D^*}^2}\left[\left(p_{1\mu}+p_{2\mu}-\frac{m_\psi^2-m_{D^*}^2}{q^2}q_\mu\right)\tilde{V}_3(q^2)\right. \\
&\left.+\frac{m_\psi^2-m_{D^*}^2}{q^2}q_\mu\tilde{V}_4(q^2)\right] \\
&-(\epsilon_\psi\cdot q)\epsilon^*_{D^*\,\mu}\tilde{V}_5(q^2)+(\epsilon^*_{D^*}\cdot q)\epsilon_{\psi\,\mu}\tilde{V}_6(q^2)\,, \qquad (5)
\end{aligned}$$

where $q=p_1-p_2$, and the convention $\mathrm{Tr}[\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma\gamma_5]=4\mathrm{i}\epsilon_{\mu\nu\rho\sigma}$ is adopted. For the transition of J/ψ into a charmed pseudoscalar meson, which is induced by the weak current, there are four independent form factors: V, A_0, A_1, A_2; while there are ten form factors for J/ψ transiting into a charmed vector meson, which are parameterized as $\tilde{A}_i$ $(i=1,2,3,4)$, $\tilde{V}_j$ $(j=1,2,3,4,5,6)$.

According to the quark diagrams in Fig. 1, the decays are classified into two categories: color allowed and suppressed processes. For the color allowed processes, the decay amplitudes are proportional to

$$a_1 = C_1 + C_2/N_c\,, \quad (6)$$

with N_c being the color number of QCD. Because $C_1\sim 1$ and $C_2\sim\alpha_s$, a_1 is estimated to be of order 1. For the decays shown in Fig. 1b, the amplitude is proportional to

$$a_2 = C_2 + C_1/N_c\,. \quad (7)$$

As $a_2/a_1\sim 1/N_c$, this kind of decays are usually called color suppressed processes.

For the decays of $c\to su\bar{d}$, which is the Cabibbo favored process, the CKM element $V_{cs}V_{ud}$ responsible for these modes is close to 1. For the Cabibbo suppressed transitions of $c\to du\bar{d}$ and $c\to su\bar{s}$, the corresponding CKM parameters $V_{cd}V_{ud}$ and $V_{cs}V_{us}$ are suppressed by a factor $\sin\theta_\mathrm{C}\approx 0.22$ with θ_C being the Cabibbo angle. The doubly suppressed processes, such as $c\to du\bar{s}$, which are suppressed by $\sin^2\theta_\mathrm{C}$, are neglected in our case. Thus, the prevailing decay modes are both color allowed and Cabibbo favored ones. The less dominant modes are the color suppressed but Cabibbo favored or color allowed but Cabibbo suppressed processes. These are the processes we will focus on in this study. Note that there are no annihilation type contributions in our case at all.

Now, we are able to write down the decay amplitudes associated with the non-leptonic two-body decays of J/ψ explicitly based on the information we achieved before. In light of the characters of final states, three different types of the processes $J/\psi\to DP$, DV, D^*P and D^*V will be investigated one by one in the following sections. For J/ψ decaying into two pseudoscalars where one is a D meson and the other is a light meson P, the decay amplitude is written as

$$\begin{aligned}
A(J/\psi\to DP) &= \langle DP|\mathcal{H}_\mathrm{eff}|J/\psi\rangle \\
&= \frac{G_\mathrm{F}}{\sqrt{2}}V^*_{cq}V_{uq'}a_i 2m_\psi(\epsilon_\psi\cdot q)f_P A_0(q^2)\,, \qquad (8)
\end{aligned}$$

where q denotes the momentum of the light emitted meson; a_i is the effective coefficients with a_1 for a color allowed process and a_2 for a color suppressed process.

The decay amplitude of $J/\psi\to DV$ decay is given as

$$\begin{aligned}
&A(J/\psi\to DV) \\
&= \frac{G_\mathrm{F}}{\sqrt{2}}V^*_{cq}V_{uq'}a_i f_V m_V\left\{-\epsilon_{\mu\nu\alpha\beta}\epsilon^{*\mu}_V\epsilon^\nu_\psi p^\alpha_\psi p^\beta_D\frac{2V(q^2)}{m_\psi+m_D}\right. \\
&+\mathrm{i}(m_\psi+m_D)(\epsilon_\psi\cdot\epsilon^*_V)A_1(q^2) \\
&\left.+\mathrm{i}\frac{(\epsilon_\psi\cdot q)[\epsilon^*_V\cdot(p_1+p_2)]}{m_\psi+m_D}2A_2(q^2)\right\}; \qquad (9)
\end{aligned}$$

for the decay amplitude of $J/\psi\to D^*P$ we have

$$\begin{aligned}
&A(J/\psi\to D^*P) \\
&= \mathrm{i}\frac{G_\mathrm{F}}{\sqrt{2}}V^*_{cq}V_{uq'}a_i f_P\left\{2\mathrm{i}\epsilon_{\mu\nu\alpha\beta}p_1^\mu p_2^\nu\epsilon^\alpha_\psi\epsilon^{*\beta}_{D^*}\tilde{A}_1(q^2)\right. \\
&+(\epsilon_\psi\cdot\epsilon^*_{D^*})\left[(m_\psi^2-m_{D^*}^2)\tilde{V}_1(q^2)-q^2\tilde{V}_2(q^2)\right] \\
&\left.+(\epsilon_\psi\cdot q)(\epsilon^*_{D^*}\cdot q)\left[-\tilde{V}_4(q^2)+\tilde{V}_5(q^2)-\tilde{V}_6(q^2)\right]\right\}. \qquad (10)
\end{aligned}$$

Lastly, the expressions for $J/\psi\to D^*V$ can be readily derived from (3) and (4):

$$\begin{aligned}
A(J/\psi\to D^*V) &= \frac{G_\mathrm{F}}{\sqrt{2}}V^*_{cq}V_{uq'}a_i f_V m_V \\
&\times\left\{-\mathrm{i}\epsilon_{\mu\nu\alpha\beta}\epsilon^\alpha_\psi\epsilon^{*\beta}_{D^*}\epsilon^{*\mu}_V\left[\left(p_1^\nu+p_2^\nu-\frac{m_\psi^2-m_{D^*}^2}{q^2}q^\nu\right)\tilde{A}_1(q^2)\right.\right.
\end{aligned}$$

$$+\frac{m_\psi^2-m_{D^*}^2}{q^2}q^\nu \tilde{A}_2(q^2)\bigg] + \frac{\mathrm{i}}{m_\psi^2-m_{D^*}^2}\epsilon_{\mu\nu\alpha\beta}p_1^\alpha p_2^\beta \epsilon_V^{*\mu}$$
$$\times\left[\tilde{A}_3(q^2)\epsilon_\psi^\nu \epsilon_{D^*}^*\cdot q - \tilde{A}_4(q^2)\epsilon_{D^*}^{*\nu}\epsilon_\psi\cdot q\right]$$
$$-(\epsilon_\psi\cdot\epsilon_{D^*}^*)\left[\epsilon_V^*\cdot(p_1+p_2)\tilde{V}_1(q^2)\right]$$
$$+\frac{(\epsilon_\psi\cdot q)(\epsilon_{D^*}^*\cdot q)}{m_\psi^2-m_{D^*}^2}\left[\epsilon_V^*\cdot(p_1+p_2)\tilde{V}_3(q^2)\right]$$
$$-(\epsilon_\psi\cdot q)\epsilon_{D^*}^*\cdot\epsilon_V^*\tilde{V}_5(q^2)+(\epsilon_{D^*}^*\cdot q)\epsilon_\psi\cdot\epsilon_V^*\tilde{V}_6(q^2)\bigg\}\,, \quad (11)$$

where ϵ_V^* denotes the polarization vector of light emitted mesons.

3 Decay rates for non-leptonic weak decays of J/ψ

3.1 Input parameters

The decay rates of the non-leptonic decays $J/\psi \to D + M$ can be written

$$\Gamma_{\psi\to DM} = \frac{1}{3}\frac{1}{8\pi}|A(J/\psi\to DM)|^2\frac{|\mathbf{p}_D|}{m_\psi^2}\,, \quad (12)$$

where $\mathbf{p}_D$ denotes the three-momentum of the final D meson in the rest frame of J/ψ and the factor "$\frac{1}{3}$" is due to the spin average of J/ψ. In order to calculate the decay rates, the input parameters, including the CKM parameters, effective Wilson coefficients, decay constants and transition form factors are necessary. The CKM parameters are taken from [21]:

$$|V_{ud}| = 0.974\,, \qquad |V_{us}| = 0.227\,,$$
$$|V_{cd}| = 0.227\,, \qquad |V_{cs}| = 0.973\,. \quad (13)$$

The effective Wilson coefficients are determined as [5]

$$a_1 = 1.26\,, \quad a_2 = -0.51\,, \quad (14)$$

which are extracted from the isospin analysis for $D \to K\pi$ decays with the help of the factorization ansatz [22].

The decay constants for light mesons are taken as [21, 23]

$$f_\pi = 0.131\,\mathrm{GeV}\,, \qquad f_K = 0.160\,\mathrm{GeV}\,,$$
$$f_\rho = 0.209\pm0.002\,\mathrm{GeV}\,, \quad f_{K^*} = 0.217\pm0.005\,\mathrm{GeV}\,, \quad (15)$$

where the pseudoscalar decay constants are experimentally determined from the combined rate for $P \to l^\pm\nu_l$ and $P \to l^\pm\nu_l\gamma$, and the vector meson longitudinal decay constants are extracted from the data on $\tau^- \to (\rho^-, K^{*-})\nu_\tau$ [21].

Besides, the values of all the relevant transition form factors $F_i(q^2)$ are taken from our earlier study [6] where the detailed expression are presented. In the literature, the form factors A_0 and A_1 have been calculated in [4, 5]; they are grouped in Table 1 together with the numbers obtained by the QCD sum rule approach [6]. We can see that the form factors at zero momentum transfer predicted in the BSW model [24] are approximately greater than that in the QCD sum rule calculation by a factor 2.

Table 1. The form factors A_0 and A_1 at $q^2 = 0$ responsible for the decays of $J/\psi \to D_{(s)}$ in the BSW model [4] and in the QCDSR [6] approach

Models	$A_0^{\psi\to D}$	$A_0^{\psi\to D_s}$	$A_1^{\psi\to D}$	$A_1^{\psi\to D_s}$
BSW	0.61	0.66	0.68	0.78
QCDSR	0.27	0.37	0.27	0.38

3.2 Branching ratios of non-leptonic decays

The numerical results of the branching ratios for the non-leptonic decays of $J/\psi \to D_{(s)}P$ are presented in Table 2, where the numbers obtained in [4, 5] are also collected together for comparison. Here, the results are given for decays, which include the charge conjugate process; for instance, $\mathrm{BR}(J/\psi \to D_s\pi)$ is the branching ratio for the decays of $J/\psi \to D_s^+\pi^- + D_s^-\pi^+$.

Table 2 shows that the decay rate for the color allowed and Cabibbo favored channel $J/\psi \to D_s\pi$ calculated in this work is 5 times smaller than that given in [5]. Such a discrepancy may be attributed to two aspects: firstly, an $\mathrm{SU}(4) f_{ijk}$ rotation matrix is employed in [5] to relate the $J/\psi \to D_s$ transition to $D \to K^*$ decay, and the form factor $A_0(0)$ is estimated as 0.7–0.8, which is almost twice that computed in the QCD sum rule approach. Secondly, the experimental result on the total decay width of J/ψ used in [5] is 67.0 keV; however, this value has been updated to 93.4 ± 2.1 keV [21].

For the Cabibbo suppressed but color allowed mode $J/\psi \to D_sK$, the relation

$$R_1 \equiv \frac{\mathrm{BR}(J/\psi\to D_sK)}{\mathrm{BR}(J/\psi\to D_s\pi)} \approx \left|\frac{V_{us}f_K}{V_{ud}f_\pi}\right|^2 \approx 0.081 \quad (16)$$

is obtained in the factorization assumption. Similarly, we can define the parameter R_2 by

$$R_2 \equiv \frac{\mathrm{BR}(J/\psi\to D\pi)}{\mathrm{BR}(J/\psi\to D_s\pi)} \approx \left|\frac{V_{cd}A_0^{\psi D}(m_\pi^2)}{V_{cs}A_0^{\psi D_s}(m_\pi^2)}\right|^2 \approx 0.032\,, \quad (17)$$

Table 2. Branching ratios of non-leptonic decays of $J/\psi \to D_{(s)}P$ (in units of 10^{-10})

	Other works	This study
$\mathrm{BR}(J/\psi\to D_s\pi)$	17.4 [4] 10.0 [5]	$2.0^{+0.4}_{-0.2}$
$\mathrm{BR}(J/\psi\to D_sK)$	1.10 [4]	$0.16^{+0.02}_{-0.02}$
$\mathrm{BR}(J/\psi\to D\pi)$	1.10 [4]	$0.080^{+0.02}_{-0.02}$
$\mathrm{BR}(J/\psi\to DK)$	–	$0.36^{+0.10}_{-0.08}$

which is also in agreement with that listed in Table 2, as long as the phase space is properly considered for these two channels.

Now, we move to the discussion of the color suppressed mode $J/\psi \to DK$. The ratio of decay rates between it and $J/\psi \to D_s\pi$ can be estimated to be

$$R_3 \equiv \frac{\mathrm{BR}(J/\psi \to DK)}{\mathrm{BR}(J/\psi \to D_s\pi)} \approx \left|\frac{a_2 A_0^{\psi D}\left(m_K^2\right)}{a_1 A_0^{\psi D_s}\left(m_\pi^2\right)}\right|^2 \approx 0.18\,, \quad (18)$$

which is consistent with that collected in Table 2. In addition, we should emphasize that the ratio R_3 is quite sensitive to the effective Wilson coefficient a_2, which can receive considerable corrections [25–29, 31–33] – for a review see [30] – due to uncertainties of the renormalization scale, higher order effects together with non-factorizable contributions, where we also refer to [20] for a recent comment.

Table 3 collects the numerical results for $J/\psi \to D^*_{(s)}P$ and $D_{(s)}V$ decays. As one can see, the branching ratio of $J/\psi \to D_s\rho$ computed in [4] is 5.8 times larger than that evaluated in this work. It is shown in [4] that the dominant contributions for the decay width of $B \to D^{(*)}P$ are from the form factor $A_1(q^2)$ corresponding to the S partial wave in the final states. As listed in Table 1, the number of the form factor A_1 derived in the BSW model is 2.1 times greater than that in terms of the QCD sum rules, which can indeed result in an enormous discrepancy for the branching fraction of $J/\psi \to D_s$ obtained in the two different approaches. Moreover, the ratio

$$R_4 \equiv \frac{\mathrm{BR}(J/\psi \to D_s\rho)}{\mathrm{BR}(J/\psi \to D_s\pi)} \quad (19)$$

is usually introduced from the viewpoint of experiment, whose value is estimated as 6.3 and 4.2, respectively, in the framework of the QCDSR and BSW model. Therefore, the decay of $J/\psi \to D_s\rho$ is more detectable than the corresponding pseudoscalar channel $J/\psi \to D_s\pi$ in experiment. Moreover, it can be seen that the ratio of the decay rates is not sensitive to the absolute magnitude of the transition form factors on account of the large cancelations of the non-perturbative effects. The relative magnitude of decay widths for the Cabibbo suppressed as well as color suppressed processes to the mode of $J/\psi \to D_s\rho$ can readily be derived by following the discussion of $J/\psi \to D_{(s)}P$ and will not be repeated.

Furthermore, we group the decay rates for the dominant channels of $J/\psi \to D^*_{(s)}V$ in Table 4, from which we can observe that $\mathrm{BR}(J/\psi \to D^*_s\rho)$ can reach 5.3×10^{-9} and that it stands as the most promising mode to be measured at BESIII. In addition, it is also helpful to define the following two ratios:

$$R_5 \equiv \frac{\mathrm{BR}(J/\psi \to D^*_s\pi)}{\mathrm{BR}(J/\psi \to D_s\pi)}\,, \quad R_6 \equiv \frac{\mathrm{BR}(J/\psi \to D^*_s\rho)}{\mathrm{BR}(J/\psi \to D_s\rho)}\,, \quad (20)$$

which characterize the relative size of the branching fractions to distinguish the final states with vector and pseudoscalar ones, respectively, in the non-leptonic two-body weak decays of J/ψ. The numbers of R_5 and R_6 are evaluated as 7.5 and 4.2 in the QCD sum rule approach, while they are determined as 3.5 and 1.4, respectively, with the ISGW model in the framework of heavy-quark spin symmetry [1]. Such discrepancies can be attributed to the different values of the form factors employed in the numerical calculations.

Moreover, we also mention that

$$R_7 \equiv \frac{\mathrm{BR}(J/\psi \to D^*_sK^*)}{\mathrm{BR}(J/\psi \to D^*\rho)}\frac{\mathrm{BR}(J/\psi \to D\rho)}{\mathrm{BR}(J/\psi \to D_sK^*)} \quad (21)$$

should be equal to 1 in the heavy-quark limit. However, this ratio is estimated to be 0.48 in the QCD sum rule approach owing to a serious suppression factor from the phase space for the decay of $J/\psi \to D^*_sK^*$ for the limited charm quark mass.

Combining Tables 2–4, we find that the branching ratio for inclusive weak decay of J/ψ can be as large as 1.3×10^{-8}, which is also in remarkable agreement with the naive estimation

$$\mathrm{BR}(J/\psi \to X_c + \dots) \approx \frac{2\Gamma_{D^\pm}}{\Gamma_{J/\psi}} \approx 1.4 \times 10^{-8}\,. \quad (22)$$

4 Discussion and conclusions

Since J/ψ mainly decays via strong and electromagnetic interactions, its weak decays usually take small fractions, which cannot be measured by any available experimental

Table 3. Branching ratios of non-leptonic decays of $J/\psi \to D^*_{(s)}P$ and $D_{(s)}V$ (in units of 10^{-10})

	Other works	This study
$\mathrm{BR}(J/\psi \to D_s\rho)$	72.6 [4]	$12.6^{+3.0}_{-1.2}$
$\mathrm{BR}(J/\psi \to D_sK^*)$	4.24 [4]	$0.82^{+0.22}_{-0.10}$
$\mathrm{BR}(J/\psi \to D\rho)$	4.40 [4]	$0.42^{+0.18}_{-0.08}$
$\mathrm{BR}(J/\psi \to DK^*)$	–	$1.54^{+0.68}_{-0.38}$
$\mathrm{BR}(J/\psi \to D^*_s\pi)$	–	$15.0^{+1.2}_{-0.4}$
$\mathrm{BR}(J/\psi \to D^*_sK)$	–	$1.1^{+0.08}_{-0.04}$
$\mathrm{BR}(J/\psi \to D^*\pi)$	–	$0.60^{+0.04}_{-0.04}$
$\mathrm{BR}(J/\psi \to D^*K)$	–	$2.6^{+0.2}_{-0.2}$

Table 4. Branching ratios of non-leptonic decays of $J/\psi \to D^*_{(s)}V$ (in units of 10^{-10})

Channels	This study
$\mathrm{BR}(J/\psi \to D^*_s\rho)$	$52.6^{+7.2}_{-6.2}$
$\mathrm{BR}(J/\psi \to D^*_sK^*)$	$2.6^{+0.4}_{-0.4}$
$\mathrm{BR}(J/\psi \to D^*\rho)$	$2.8^{+0.6}_{-0.4}$
$\mathrm{BR}(J/\psi \to D^*K^*)$	$9.6^{+3.2}_{-2.2}$

apparatus. Another aspect, however, is that because J/ψ contains two heavy constituents, its weak decay may possess a unique character. Indeed, weak decays of J/ψ may offer an ideal platform to examine the mechanism that governs the hadronization process, without possible contamination from the light spectator; also one may determine its fundamental parameters such as the CKM matrix, which can be a complementary test to the values obtained in D decays. It is lucky for high energy physicists that a tremendous database on J/ψ will be available in the forthcoming BESIII and the measurements on the weak decays of J/ψ may become possible.

As is well known, the essential challenge in the theoretical calculations on the rates of weak decays of J/ψ is to disentangle the underlying weak-interaction transitions from the notorious effects owing to strong interactions in a reasonable way [35]. In our previous paper [6], the transition form factors in the semi-leptonic weak decays of J/ψ have been investigated to the leading order of α_s based on QCD sum rules, where the non-perturbative QCD dynamics is characterized by a few universal parameters. The branching ratios for dominant exclusive processes are evaluated and their order of magnitude is typically at 10^{-10}. Obviously based on the factorization assumption, the form factors obtained for the semi-leptonic decays can be applied to study the non-leptonic decays.

This paper can be viewed as a continuation of our earlier work [6]. We present a comprehensive study of non-leptonic decays of $J/\psi \to D_{(s)} + M$ based on the factorization assumption and apply the transition form factors calculated in the QCD sum rules. It is observed that the sum of the branching fractions for the dominant non-leptonic decays of $J/\psi \to D_s^- \pi$, $D_s^- \rho$, $D_s^{*-} \pi$, and $D_s^{*-} \rho$ as well as their charge conjugate channels can reach values as large as 0.82×10^{-8}, the special decay mode $J/\psi \to D_s^* \rho$ can even arrive at 5.3×10^{-9}, which one may hope to marginally detect in the e^+e^- colliders in view of the large database of the BESIII. Our results are in agreement with the finding in [1] that J/ψ decays to vector charmed meson D_s^{*-} more favorably than to the pseudoscalar one; however, the ratios of these two channels calculated in this work are twice or three times larger than that given by [1], where heavy-quark spin symmetry and the non-recoil approximation were adopted and the ISGW model was employed to compute the single form factor η_{12}.

As one may see a possibility of measuring weak decays of J/ψ, which have obvious advantages for getting insight into the physics picture, we strongly urge our experimental colleagues to search for vector charmed mesons productions in J/ψ decays at BESIII [2, 3], although it is a great challenge to observe the non-leptonic decays of J/Ψ resonances from the experimental side.

Moreover, from the theoretical side, it should be emphasized that Coulomb type corrections for the heavy-quarkonium system [36–39] are not included in the computations of form factors in the QCD sum rules, which could induce additional uncertainties to the evaluation of the branching fractions for non-leptonic two-body decays of J/ψ. However, one can still trust the order of magnitude gained in this work, since while calculating the form factors that need to deal with the three-point correlations most uncertainties originating from Coulomb-like corrections are canceled by that in the two-point correlations for evaluating the decay constant of J/ψ. Apart from the weak decays of J/ψ presented in this paper, we seriously suggest to explore weak decays of Υ in a complementary fashion from both the theoretical and experimental point of view.

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